Development of identification method for minimal set of inertial parameters of multi-body system

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ABSTRACT

In motion control and motion simulation, inertial properties (mass, center of gravity, inertia tensor) of object are fundamental and important parameters that serve as input values and have a significant impact on the accuracy of results. For this reason, an accurate identification method of inertial properties is strongly demanded. In multi-body dynamics, the inertial properties of individual links cannot be identified from link motion and inter-joint torque or external force data, because they are redundant to the multi-body dynamics model. Therefore, the minimum dynamic parameters necessary to represent the multibody dynamics model has been defined and identified. These dynamic parameters are obtained by combining the geometric parameters and the inertial properties of the counterpart elements and are called the minimal set of inertial parameters. The conventional identification methods use a set of measured link motion and ground reaction forces. Only the minimal set of inertial parameters for a sagittal plane can be identified from movements such as walking motion of human bodies. Thus, it is difficult to apply these methods to the identification of individual human bodies. In this paper, a new method for identifying the minimal set of inertial parameters of a multi-body system is developed by expanding and applying the identification method based on free vibration measurements, which is the identification method for inertial properties of single-body. This method shows that all minimal set of inertial parameters can be identified with high accuracy from relatively simple motion measurements, both theoretically and by means of basic experiments.

Keywords: Minimal set of inertial parameters, Inertial properties, Identification, Motion simulation, Multi-body system.

1 INTRODUCTION

In motion control and motion simulation, inertial properties (mass, center of gravity, inertia tensor) of object are fundamental and important parameters that serve as input values and have a significant impact on the accuracy of results, therefore an accurate identification method of inertial properties is strongly demanded. Methods for identifying inertial properties can be classified into two types: theoretical calculation using CAD and other computational methods, and experimental identification with measurement data. Experimental identification is also important as a means of verifying the results of theoretical calculation methods. For this reason, various methods have been developed for the identification of the inertial properties of single-body. In particular, identification methods based on free vibration measurements [1] have been developed in recent years, making it possible to measure inertial properties with sufficiently high accuracy and short measurement time for practical use.

In robotics and ergonomics, geometric parameters such as link lengths and the inertial properties of individual links are required when dealing with multi-body dynamics models consisting of multiple parts connected by joints. Furthermore, there is a demand to identify the inertial properties of the links in the connected state, due to the identification requirements of the mechanism such as robot in the assembled state and the constraint that the body cannot naturally be detached in the measurement of the human body. While geometric parameters can be easily identified from static measurements, the inertial properties of individual links cannot be identified from link motion and inter-joint torque or external force data, because they are redundant to the multi-body dynamics model. Therefore, the minimum dynamic parameters necessary to represent the multibody dynamics model has been defined and identified. These dynamic parameters are obtained by combining the geometric parameters and the inertial properties of the counterpart elements and are called the minimal set of inertial parameters.

Numerical and analytical methods for calculating the minimal set of inertial parameters have been established [2-4] and various experimental identification methods have been developed. The conventional identification methods utilize a set of measured link motion and inter-joint torques [5] or a set of measured link motion and floor reaction forces [6]. The former requires a torque sensor for every joint. Thus, it cannot be used in cases where it is difficult to estimate or measure joint torque, such as humans and humanoid robots. The latter can only identify the minimal set of inertial parameters for a plane in the direction of travel from movements such as walking. It has problems such as the difficulty in selecting an effective motion for identifying all minimal set of inertial parameters and the large measurement error and has not yet applied to the identification of individual human bodies. Thus, a new identification method which can be applied to individual human bodies is demanded.

In the previous method for identifying inertial properties by free vibration measurement [1], the free vibration of an object with small amplitude is measured under the pseudo-peripheral free boundary condition with suspension springs. The effect of the suspension spring is precisely modelled and considered to enable highly accurate identification. By expressing the forces on the suspension spring in three dimensions in terms of a stiffness matrix, if either the force or deformation on the suspension spring is measured, it is possible to calculate the other. The advantage of this method is that the forces can be calculated by measuring the position and posture of the platform suspended by suspension springs, and no actuator or load cell is required. However, this method is the identification method for single-body and cannot simply be applied to the identification of the minimal set of inertial parameters of multi-body system.

Based on this background, in this paper, a new method for identifying the minimal set of inertial parameters of multi-body system is developed by expanding and applying the identification method based on free vibration measurements, which is a method for identifying the inertial properties of a single-body, and evaluated its performance.

2 IDENTIFICATION METHOD FOR MINIMAL SET OF INERTIAL PARAMETERS

In our new method, the procedure for identifying the minimal set of inertial parameters of a multi-body system consists of two steps. As shown in Fig. 1, the multibody system is modelled as a base link and other links, with the base link attached to the platform and the platform connected to the suspension springs. In the first step, the relative motion between the links is fixed to set a posture and the overall inertial properties in several postures are identified by using identification method for the single-body inertial properties [1]. In the second step, the relative motion between the links is measured. This motion is caused by muscular forces when human body is measured.

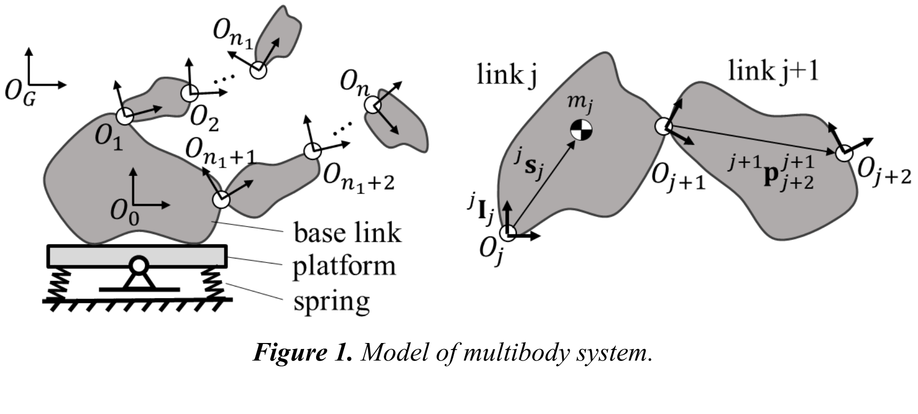

Figure 1. *Model of multibody system.*

2.1 Identification of the overall inertial properties in several postures

In the following, a multi-body system connected by spherical joints is taken as an example for the formulation. Equations (1)-(3) show the minimal set of inertial parameters of a multi-body system connected by spherical joints [3].

$$M_{j-1} = m_{j-1} + M_j, \tag{1}$$

$$\mathbf{MS}_{j-1} = m_{j-1}\,{}^{j-1}\mathbf{s}_{j-1} + M_j\,{}^{j-1}\mathbf{p}_j^{j-1}, \tag{2}$$

$$\mathbf{J}_{j-1} = {}^{j-1}\mathbf{I}_{j-1} + M_j\left[{}^{j-1}\mathbf{p}_j^{j-1}\times\right]^T\left[{}^{j-1}\mathbf{p}_j^{j-1}\times\right], \tag{3}$$

where the subscript j indicates the j-th link($0 \leqq j \leqq n$), m_j is the mass of link j, ${}^j\mathbf{s}_j$ is the vector from the j-coordinate to the center of gravity of link j, ${}^j\mathbf{I}_j$ is the inertia tensor of link j around the j-coordinate origin, expressed in the j-coordinate system, ${}^{j-1}\mathbf{p}_j^{j-1}$ is the vector from the j-1 coordinate origin to the j-coordinate origin, expressed in the j-1 coordinate system, M_j is the minimal set of inertial parameters for the mass of link j, $\mathbf{MS}_j$ is the minimal set of inertial parameters for the center of gravity of link j and $\mathbf{J}_j$ is the minimal set of inertial parameters for the inertia tensor of link j. The operator $[\ \times]$ is defined by Equation (4).

$$\left[{}^{j-1}\mathbf{p}_j^{j-1}\times\right] \triangleq \begin{bmatrix} 0 & -{}^{j-1}\mathbf{p}_{jz}^{j-1} & {}^{j-1}\mathbf{p}_{jy}^{j-1} \\ {}^{j-1}\mathbf{p}_{jz}^{j-1} & 0 & -{}^{j-1}\mathbf{p}_{jx}^{j-1} \\ -{}^{j-1}\mathbf{p}_{jy}^{j-1} & {}^{j-1}\mathbf{p}_{jx}^{j-1} & 0 \end{bmatrix}. \tag{4}$$

The overall inertial properties in several postures are identified by using identification method for the single-body inertial properties [1]. Equations (5)-(6) show the relationship between the overall inertial properties and the minimal set of inertial parameters of each link.

$$M_0\mathbf{s}_{t,i} = \textstyle\sum_{j=0}^{n}\ {}^0\mathbf{R}_{j,i}\mathbf{MS}_j, \tag{5}$$

$$\begin{aligned} \mathbf{I}_{tg,i} - M_0\left[\mathbf{s}_{t,i}\times\right]^T\left[\mathbf{s}_{t,i}\times\right] &= \mathbf{J}_0 - \left[\mathbf{s}_{t,i}\times\right]^T\left[\textstyle\sum_{j=0}^{n}\ {}^0\mathbf{R}_{j,i}\mathbf{MS}_j\times\right] - \left[\textstyle\sum_{j=0}^{n}\ {}^0\mathbf{R}_{j,i}\mathbf{MS}_j\times\right]^T\left[\mathbf{s}_{t,i}\times\right] \\ &+ \textstyle\sum_{j=1}^{n}\Big({}^0\mathbf{R}_{j,i}\mathbf{J}_j\,{}^0\mathbf{R}_{ji}^T + \left[\textstyle\sum_{k=1}^{j}({}^0\mathbf{R}_{j-1,i}\,{}^{j-1}\mathbf{p}_j^{j-1})\times\right]^T\left[{}^0\mathbf{R}_{j,i}\mathbf{MS}_j\times\right] \\ &+ \left[{}^0\mathbf{R}_{j,i}\mathbf{MS}_j\times\right]^{\boldsymbol{T}}\left[\textstyle\sum_{k=1}^{j}({}^0\mathbf{R}_{j-1,i}\,{}^{j-1}\mathbf{p}_j^{j-1})\times\right]\Big), \end{aligned} \tag{6}$$

where the subscript i indicates the i-th measurement, $\mathbf{s}_t$ is the vector from the base link coordinates to the center of gravity of multi-body system, expressed in the base link coordinate system, $\mathbf{I}_{tg}$ is the inertia tensor of multi-body system around the center of gravity, expressed in the base link coordinate system and ${}^0\mathbf{R}_j$ is the rotation matrix that transforms from the j-coordinate system to the base link coordinate system. M_0 indicates the overall mass and is treated as a known parameter as the overall weight can be measured.

The overall inertial properties are identified for k postures. Using Equations (5) and (6), the results of each identification can be arranged and expressed as a linear relationship for the minimal set of inertial parameters, as in Equation (7).

$$\begin{bmatrix} M_0 s_{tx,1} \\ M_0 s_{ty,1} \\ M_0 s_{tz,1} \\ I_{tgxx,1} - M_0\left(s_{ty,1}^2 + s_{tz,1}^2\right) \\ I_{tgyy,1} - M_0\left(s_{tx,1}^2 + s_{tz,1}^2\right) \\ I_{tgzz,1} - M_0\left(s_{tx,1}^2 + s_{ty,1}^2\right) \\ I_{tgxy,1} + M_0 s_{tx,1}s_{ty,1} \\ I_{tgzx,1} + M_0 s_{tx,1}s_{tz,1} \\ I_{tgyz,1} + M_0 s_{ty,1}s_{tz,1} \\ \vdots \\ I_{tgyz,k} + M_0 s_{ty,1}s_{tz,k} \end{bmatrix} = \mathbf{Z}\mathbf{\Phi}_B, \tag{7}$$

$$\boldsymbol{\Phi}_B = [\boldsymbol{\Phi}_{B0}^T, \boldsymbol{\Phi}_{B1}^T, \cdots, \boldsymbol{\Phi}_{Bn}^T]^T, \tag{8}$$

$$\boldsymbol{\Phi}_{Bj} = \left[MS_{jx}, MS_{jy}, MS_{jz}, J_{jxx}, J_{jyy}, J_{jzz}, J_{jyz}, J_{jzx}, J_{jxy}\right]^T, \tag{9}$$

where $\boldsymbol{\Phi}_B \in \mathbb{R}^{9j\times 1}$ is a vector of minimal set of inertial parameters and $\mathbf{Z} \in \mathbb{R}^{9k\times 9j}$ is a coefficient matrix over $\boldsymbol{\Phi}_B$, consisting of rotation matrix $\mathbf{R}$ and the vector $\mathbf{p}$.

The rank of the coefficient matrix $\mathbf{Z}$ is less than the number of minimal sets of inertial parameters, no matter how many times the overall inertial properties are measured by changing the posture. Therefore, $\boldsymbol{\Phi}_B$ cannot be obtained from Equation (7) directly.

Then in this paper, Equation (7) is transformed as follows,

$$\begin{bmatrix} M_0 s_{tx,1} \\ M_0 s_{ty,1} \\ M_0 s_{tz,1} \\ I_{tgyy,1} - M_0(s_{tx,1}^2 + s_{tz,1}^2) \\ I_{tgxx,1} - M_0\left(s_{ty,1}^2 + s_{tz,1}^2\right) - I_{tgyy,1} + M_0(s_{tx,1}^2 + s_{tz,1}^2) \\ I_{tgxx,1} - M_0\left(s_{ty,1}^2 + s_{tz,1}^2\right) - I_{tgzz,1} + M_0(s_{tx,1}^2 + s_{ty,1}^2) \\ I_{tgxy,1} + M_0 s_{tx,1} s_{ty,1} - I_{tgzx,1} - M_0 s_{tx,1} s_{tz,1} \\ I_{tgzx,1} + M_0 s_{tx,1} s_{tz,1} - I_{tgyz,1} - M_0 s_{ty,1} s_{tz,1} \\ I_{tgxy,1} + M_0 s_{tx,1} s_{ty,1} + I_{tgzx,1} + M_0 s_{tx,1} s_{tz,1} \\ \vdots \\ I_{tgxy,k} + M_0 s_{tx,k} s_{ty,k} + I_{tgzx,k} + M_0 s_{tx,k} s_{tz,k} \end{bmatrix} = \mathbf{Z}_s \boldsymbol{\Phi}_s, \tag{10}$$

$$\boldsymbol{\Phi}_s = \left[\boldsymbol{\Phi}_{s0}^T, \boldsymbol{\Phi}_{s1}^T, \cdots, \boldsymbol{\Phi}_{sn}^T, \textstyle\sum_{j=0}^{n} J_{jyy}\right]^T, \tag{11}$$

$$\boldsymbol{\Phi}_{sj} = \left[MS_{jx}, MS_{jy}, MS_{jz}, J_{jxx} - J_{jyy}, J_{jzz} - J_{jyy}, J_{jyz}, J_{jzx}, J_{jxy}\right]^T, \tag{12}$$

where $\boldsymbol{\Phi}_s \in \mathbb{R}^{8j+1\times 1}$ is a subset vector of minimal set of inertial parameters and consists of a part of minimal set of inertial parameters and relative expression for the diagonal term of the inertia tensor and $\mathbf{Z}_s \in \mathbb{R}^{9k\times 8j+1}$ obtained by transforming $\mathbf{Z}$ is a coefficient matrix over $\boldsymbol{\Phi}_s$.

By identifying the overall inertial properties in multiple postures, it is possible to identify the subset vector $\boldsymbol{\Phi}_s$ from Equation (10) when the coefficient matrix $\mathbf{Z}_s$ is of sufficient rank to identify the subset vector $\boldsymbol{\Phi}_s$.

2.2 Measurement of the relative motion between links

The identification of the overall inertial properties in multiple postures yields the subset vector $\boldsymbol{\Phi}_s$. As a result, the entire minimal set of inertial parameters can be obtained if the remaining J_{jyy} can be obtained. The value of J_{jyy} can be identified from relatively simple movements, such as the relative motion of links on a certain plane. In experiments, relative motion between the links can be easily measured.

The equations of motion of the multi-body system can be expressed by the model shown in Equation (13) [7]. The upper part of Equation (13) represents the free motion of the base link, while the lower part represents the chain motion of the links connected to the base link.

$$\begin{bmatrix} \mathbf{H}_{O1} & \mathbf{H}_{O2} \\ \mathbf{H}_{C1} & \mathbf{H}_{C2} \end{bmatrix} \begin{bmatrix} \ddot{\mathbf{q}}_0 \\ \ddot{\mathbf{q}}_C \end{bmatrix} + \begin{bmatrix} \mathbf{b}_O \\ \mathbf{b}_C \end{bmatrix} = \begin{bmatrix} 0 \\ \boldsymbol{\tau} \end{bmatrix} + \begin{bmatrix} \mathbf{F}_O \\ \mathbf{F}_C \end{bmatrix}, \tag{13}$$

where $\mathbf{H}$ is the inertia matrix, $\mathbf{b}$ is the sum of the Coriolis force, centrifugal force and gravity force, $\boldsymbol{\tau}$ is the joint torque, $\mathbf{F}$ is the external force, $\mathbf{q}_0$ is the generalized coordinates of the base link and $\mathbf{q}_C$ is the joint angle vector.

Equation (13) can be transformed into Equation (14) using a vector of aligned minimal set of inertial parameters $\boldsymbol{\Phi}_B$ [7].

$$\begin{bmatrix} \mathbf{Y}_{B1} \\ \mathbf{Y}_{B2} \end{bmatrix} \boldsymbol{\Phi}_B + \begin{bmatrix} \mathbf{Y}_{B1M0} \\ \mathbf{Y}_{B2M0} \end{bmatrix} M_0 = \begin{bmatrix} 0 \\ \boldsymbol{\tau} \end{bmatrix} + \begin{bmatrix} \mathbf{F}_O \\ \mathbf{F}_C \end{bmatrix}, \tag{14}$$

where $\mathbf{Y}$ is called as the regressor matrix over $\boldsymbol{\Phi}_B$ and is function of $\mathbf{q}_0$, $\mathbf{q}_C$, $\dot{\mathbf{q}}_0$, $\dot{\mathbf{q}}_C$, $\ddot{\mathbf{q}}_0$ and $\ddot{\mathbf{q}}_C$.

The lower part of Equation (14) is utilized in the identification methods using a set of measured link motion and inter-joint torques [5]. The upper part of Equation (14) and floor reaction forces as external forces is utilized in the identification methods a set of measured link motion and floor reaction forces [6]. Our proposed method uses the upper part of Equation (14). The external force is replaced the forces received from the suspension springs of the measurement system installed to create the boundary conditions of pseudo peripheral freedom, which can be calculated from the position measurements because they are modelled precisely. Measurement of the joint torque $\boldsymbol{\tau}$ is not required in our proposed method. As the base link is fixed to the platform, the influence of the platform is also considered.

$$\mathbf{Y}_{B1}\mathbf{\Phi}_B + \mathbf{Y}_{B1M0}M_0 + \mathbf{M}_P\ddot{\mathbf{q}}_0 = \mathbf{K}\mathbf{q}_0, \tag{15}$$

where $\mathbf{M}_P$ is the mass matrix of the platform and $\mathbf{K}$ is the stiffness matrix related to the forces received from the suspension springs [1].

Arranging Equation (15) at any given time yields Equation (16). Equation (16) can be used to identify the remaining J_{jyy}.

$$\begin{bmatrix} \mathbf{K}\mathbf{q}_{0,t1} - \mathbf{Y}_{B1M0,t1}M_0 - \mathbf{M}_P\ddot{\mathbf{q}}_{0,t1} \\ \mathbf{K}\mathbf{q}_{0,t2} - \mathbf{Y}_{B1M0,t2}M_0 - \mathbf{M}_P\ddot{\mathbf{q}}_{0,t2} \\ \vdots \\ \mathbf{K}\mathbf{q}_{0,tl} - \mathbf{Y}_{B1M0,tl}M_0 - \mathbf{M}_P\ddot{\mathbf{q}}_{0,tl} \end{bmatrix} = \begin{bmatrix} \mathbf{Y}_{B1,t1} \\ \mathbf{Y}_{B1,t2} \\ \vdots \\ \mathbf{Y}_{B1,tl} \end{bmatrix} \mathbf{\Phi}_B, \tag{16}$$

where the subscript tl indicates the l-th arbitrary time.

3 EXPERIMENT

3.1 Identification Result

The performance of the proposed identification method was evaluated with experiments. The object to be measured and the coordinate system are shown in Fig. 2. The measuring devices used for the measurements are shown in Fig. 3.

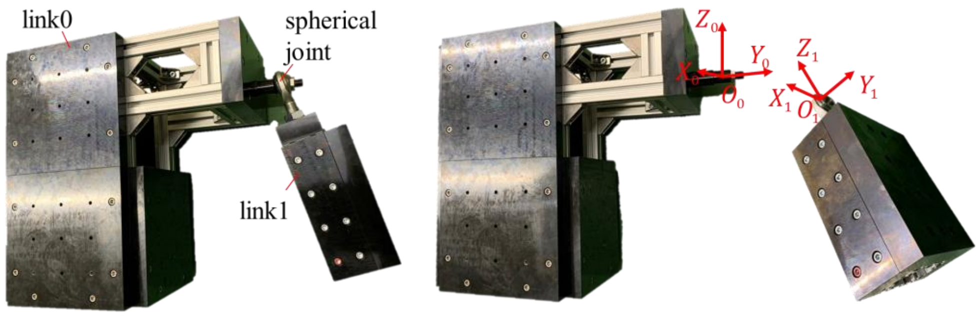

Figure 2. *Two-link measurement objects and coordiinate system.*

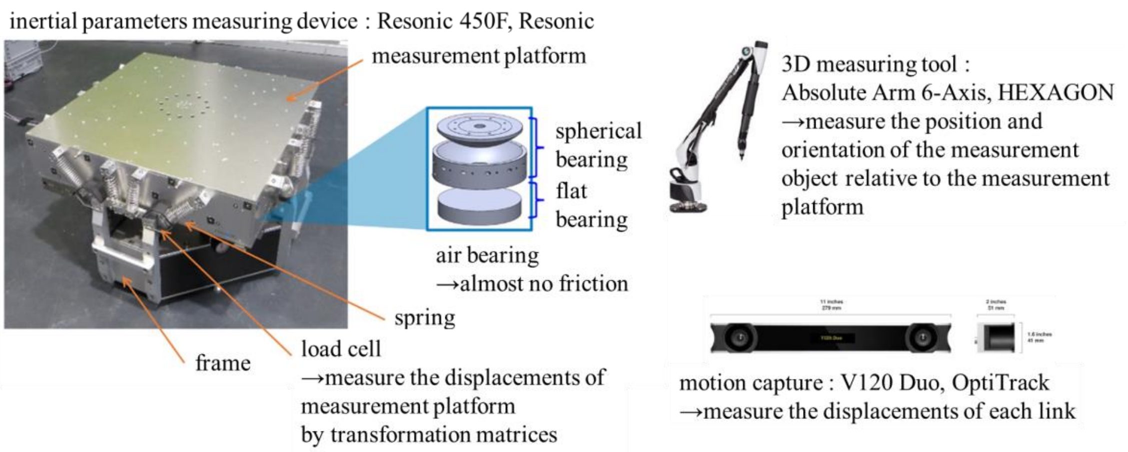

Figure 3. *Measuring tools.*

The measurement object is a system of two links connected by a spherical joint. The performance evaluation is carried out as follows. The inertial properties of each link are identified using the device shown in Fig. 3 separately and shown in Table 1. Reference values of the minimal set of inertial parameters are calculated from the separately identified results. These values are compared with the identification results of the minimal set of inertial parameters with the links connected using the proposed identification method.

Table 1. *Inertial properties of each link*

	m [kg]	I_{xx}[kgm^2]	I_{yy}[kgm^2]	I_{zz}[kgm^2]	I_{xy}[kgm^2]
link 0	65.921	1.8506	1.4892	1.4157	-0.0081
link 1	14.896	0.0774	0.1020	0.0458	0.0000
	I_{zx}[kgm^2]	I_{yz}[kgm^2]	s_x[m]	s_y[m]	s_z[m]
link 0	-0.0024	-0.1659	0.0000	-0.3072	-0.1152
link 1	0.0000	0.0041	0.0000	0.0217	-0.1747

First, the overall inertial properties are identified for multiple postures. The overall inertial properties were identified for three postures, as shown in Fig. 4. Table 2 shows the preliminary identification results and elements of the rotation matrix of each posture.

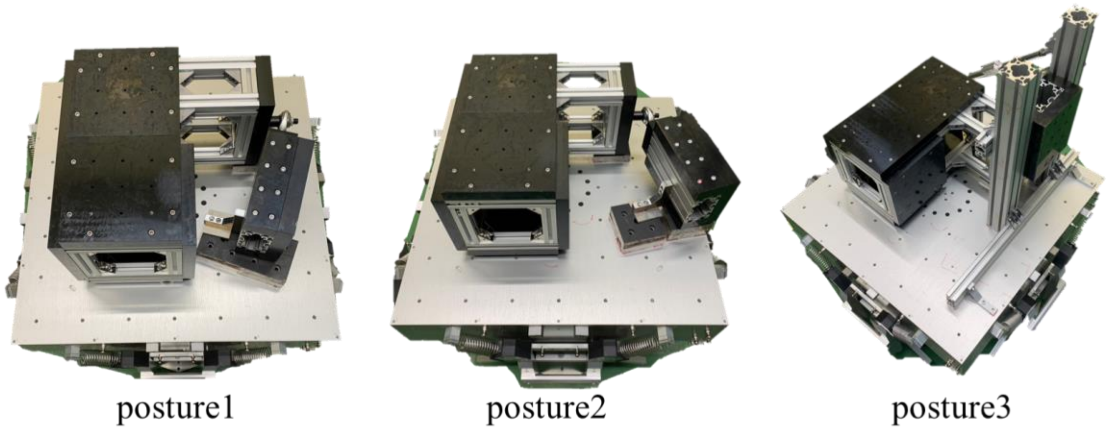

Figure 4. *Postures for identification.*

Table 2. *Preliminary identification results for inertial properties of whole body*

	posture1	posture2	posture3	${}^{0}\mathbf{R}_{1,1}$		
M_0 [kg]	80.817	80.817	80.817	0.9999	0.0071	0.0099
I_{tgxx}[kgm^2]	2.9221	3.6841	3.3737	-0.0096	0.9576	0.2880
I_{tgyy}[kgm^2]	1.6368	1.6167	2.1047	-0.0074	-0.2881	0.9576
I_{tgzz}[kgm^2]	2.3958	3.1638	3.1348	${}^{0}\mathbf{R}_{1,2}$		
I_{tgxy}[kgm^2]	0.0066	0.0155	-0.6960	0.9998	0.0037	0.0179
I_{tgzx}[kgm^2]	-0.0017	0.0000	-0.2345	0.0014	0.9620	-0.2730
I_{tgyz}[kgm^2]	0.0203	0.0723	-0.6275	-0.0182	0.2730	0.9618
s_{tgx}[m]	0.0002	-0.0001	0.0332	${}^{0}\mathbf{R}_{1,3}$		
s_{tgx}[m]	-0.2562	-0.2382	-0.2468	-0.0010	0.0000	-1.0000
s_{tgx}[m]	-0.1265	-0.1244	-0.0942	-0.0316	0.9995	0.0000
				0.9995	0.0316	-0.0010

In the proposed identification method, to obtain the subset vector $\mathbf{\Phi}_s$, an evaluation function E_{v1} is defined using Equation (10) and preliminary identified values in Table 2.

$$E_{v1} = norm\left(\begin{bmatrix} M_0 s_{tx,1} \\ M_0 s_{ty,1} \\ M_0 s_{tz,1} \\ I_{tgyy,1} - M_0(s_{tx,1}^2 + s_{tz,1}^2) \\ I_{tgxx,1} - M_0(s_{ty,1}^2 + s_{tz,1}^2) - I_{tgyy,1} + M_0(s_{tx,1}^2 + s_{tz,1}^2) \\ I_{tgxx,1} - M_0(s_{ty,1}^2 + s_{tz,1}^2) - I_{tgzz,1} + M_0(s_{tx,1}^2 + s_{ty,1}^2) \\ I_{tgxy,1} + M_0 s_{tx,1} s_{ty,1} - I_{tgzx,1} - M_0 s_{tx,1} s_{tz,1} \\ I_{tgzx,1} + M_0 s_{tx,1} s_{tz,1} - I_{tgyz,1} - M_0 s_{ty,1} s_{tz,1} \\ I_{tgxy,1} + M_0 s_{tx,1} s_{ty,1} + I_{tgzx,1} + M_0 s_{tx,1} s_{tz,1} \\ \vdots \\ I_{tgxy,3} + M_0 s_{tx,3} s_{ty,3} + I_{tgzx,3} + M_0 s_{tx,3} s_{tz,3} \end{bmatrix} - \mathbf{Z}_s \mathbf{\Phi}_s\right). \quad (17)$$

The identification is performed with an optimization calculation using the subset vector $\mathbf{\Phi}_s$ as a variable and the evaluation function E_{v1}. In this paper, the interior point method was used as the optimization method. The identification results for the subset vector $\mathbf{\Phi}_s$ are shown in Table 3.

Table 3. *Identification results for the subset vector* $\mathbf{\Phi}_s$

MS_{0x} [kgm]	MS_{0y} [kgm]	MS_{0z} [kgm]	MS_{1x} [kgm]	MS_{1y} [kgm]	MS_{1z} [kgm]
0.0599	-20.2678	-7.6113	-0.0177	0.3212	-2.6268
J_{0yz}[kgm^2]	J_{0zx}[kgm^2]	J_{0xy}[kgm^2]	J_{1yz}[kgm^2]	J_{1zx}[kgm^2]	J_{1xy}[kgm^2]
-2.5027	0.0121	0.0192	0.0519	-0.0087	-0.0052
$J_{0xx} - J_{0yy}$ [kgm^2]	$J_{0zz} - J_{0yy}$ [kgm^2]	$J_{1xx} - J_{1yy}$ [kgm^2]	$J_{1zz} - J_{1yy}$ [kgm^2]	$J_{0yy} + J_{1yy}$ [kgm^2]	
6.6079	5.2583	-0.0238	-0.5064	2.9315	

From the results shown in Table 3 and Equation (18), when J_{0yy} is obtained, all values of $\mathbf{\Phi}_B$ can be obtained.

$$\mathbf{\Phi}_B = \begin{bmatrix} \mathbf{MS}_0 \\ J_{0xx} - J_{0yy} \\ 0 \\ J_{0zz} - J_{0yy} \\ J_{0yz} \\ J_{0zx} \\ J_{0xy} \\ \mathbf{MS}_1 \\ J_{1xx} - J_{1yy} + J_{0yy} + J_{1yy} \\ J_{0yy} + J_{1yy} \\ J_{1zz} - J_{1yy} + J_{0yy} + J_{1yy} \\ J_{1yz} \\ J_{1zx} \\ J_{1xy} \end{bmatrix} + \begin{bmatrix} \mathbf{0} \\ J_{0yy} \\ J_{0yy} \\ J_{0yy} \\ 0 \\ 0 \\ 0 \\ \mathbf{0} \\ -J_{0yy} \\ -J_{0yy} \\ -J_{0yy} \\ 0 \\ 0 \\ 0 \end{bmatrix}. \quad (18)$$

Second, the relative motion between the links was measured experimentally to identify J_{0yy}. The motion was measured at a frame rate of 120 Hz using motion capture with a total of 12 markers on link 0 and the platform and 8 markers on link 1. Link 1 was tilted from its stationary position and attached to link 0 using a nylon thread, and relative motion was generated by gravity by releasing the nylon thread. The translational displacement $\mathbf{q}_o^G$ of link 0 and the rotational displacement $\boldsymbol{\theta}^G$ of link 0 and link 1, expressed in the ground-coordinate system, were obtained from the marker position data at each sampling time using the singular value decomposition method [8]. The rotational displacement $\boldsymbol{\theta}^G$ was defined in XYZ-Eulerian angles. The inertial parameters measuring device has two degrees of freedom in XY axis translation and three degrees of freedom in rotation. The velocity $\dot{\mathbf{q}}_0^G$, acceleration $\ddot{\mathbf{q}}_0^G$, angular velocity $\boldsymbol{\omega}^G$ and angular acceleration $\dot{\boldsymbol{\omega}}^G$ were obtained from this time series data using a zero phase filter with filter order 50, passband frequency 5 Hz and stopband frequency 10 Hz. Figure 5 shows these results.

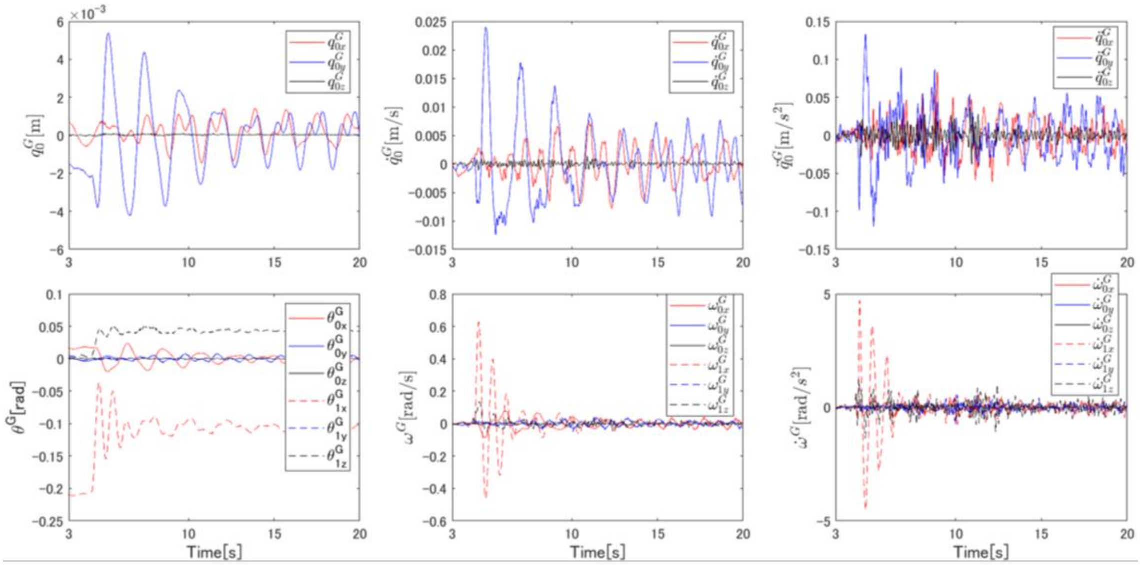

Figure 5. *Results of motion capture measurements and their derivatives.*

In addition, since the diagonal term of the inertia tensor should be positive, we obtain the following relationship with respect to J_{0yy},

$$\max\left(0, J_{0yy} - J_{0xx}, J_{0yy} - J_{0zz}\right) < J_{0yy} < \min\left(J_{0yy} + J_{1yy}, J_{0yy} + J_{1xx}, J_{0yy} + J_{1zz}\right). \quad (19)$$

To identify J_{0yy} properly, an evaluation function E_{v2} was defined using Equation (16) as shown in Equation (20).

$$E_{v2} = norm\left(\begin{bmatrix} \mathbf{K}\mathbf{q}_{0,t1} - \mathbf{Y}_{B1M0,t1}M_0 - \mathbf{M}_P\ddot{\mathbf{q}}_{0,t1} \\ \mathbf{K}\mathbf{q}_{0,t2} - \mathbf{Y}_{B1M0,t2}M_0 - \mathbf{M}_P\ddot{\mathbf{q}}_{0,t2} \\ \vdots \\ \mathbf{K}\mathbf{q}_{0,tl} - \mathbf{Y}_{B1M0,tl}M_0 - \mathbf{M}_P\ddot{\mathbf{q}}_{0,tl} \end{bmatrix} - \begin{bmatrix} \mathbf{Y}_{B1,t1} \\ \mathbf{Y}_{B1,t2} \\ \vdots \\ \mathbf{Y}_{B1,tl} \end{bmatrix} \boldsymbol{\Phi}_B\right). \quad (20)$$

The vector $\boldsymbol{\Phi}_B$ in Equation (20) is determined from Equation (18) given J_{0yy}. The measurement data from 3 to 10 seconds in Fig. 5 are adopted in Equation (20). The identification is performed with an optimization calculation using J_{0yy} as a variable and the evaluation function E_{v2}. The relationship $0 < J_{0yy} < 2.4250$ can be obtained from Equation (19) based on the identification results of $\boldsymbol{\Phi}_S$, so an optimization calculation is performed within this range. The identification results for all minimal set of inertial parameters and the results calculated from the separately identified inertial properties are shown in Table 4.

3.2 Discussion

Figure 6 shows the left side of Equation (16) with a solid line and the result of the right side calculated using the identification result with a dotted line as time series data. The solid and dotted lines are in good agreement, indicating that the optimization calculation was satisfactory.

The accuracy of the identification of the inertial properties by the measurement devices used in the experiment is $\pm$ 1 mm with respect to the center of gravity position and $\pm$ 1% of the maximum principal moment of inertia with respect to the inertia tensor.

The minimal set of inertial parameter for the center of gravity position was devided by the mass for comparison with single-body identification accuracy. The maximum errors of identification results are shown in Table 5.

Table 4. *Minimal set of inertial parameters of each link*

	M_0[kg]	J_{0xx} [kgm^2]	J_{0yy}[kgm^2]	J_{0zz}[kgm^2]	J_{0xy}[kgm^2]
ID	80.817	8.9788	2.3708	7.6291	0.0192
Cal.	80.817	8.9460	2.3639	7.6364	-0.0072
Error	0.0000	0.0328	0.0069	-0.0073	0.0264
	J_{0zx}[kgm^2]	J_{0yz}[kgm^2]	MS_{0x}[kgm]	MS_{0y}[kgm]	MS_{0z}[kgm]
ID	0.0121	-2.5027	0.0599	-20.2678	-7.6113
Cal.	-0.0020	-2.4986	0.0031	-20.2502	-7.5935
Error	0.0141	-0.0041	0.0568	-0.0176	-0.0178
	J_{1xx} [kgm^2]	J_{1yy}[kgm^2]	J_{1zz}[kgm^2]	J_{1xy}[kgm^2]	J_{1zx}[kgm^2]
ID	0.5368	0.5607	0.0542	-0.0052	-0.0087
Cal.	0.5388	0.5564	0.0528	0.0000	0.0003
Error	-0.0020	0.0043	0.0014	-0.0052	-0.0090
	J_{1yz}[kgm^2]	MS_{1x}[kgm]	MS_{1y}[kgm]	MS_{1z}[kgm]	
ID	0.0519	-0.0177	0.3212	-2.6268	
Cal	0.0606	0.0013	0.3234	-2.6017	
Error	-0.0087	-0.0190	-0.0022	-0.0251	

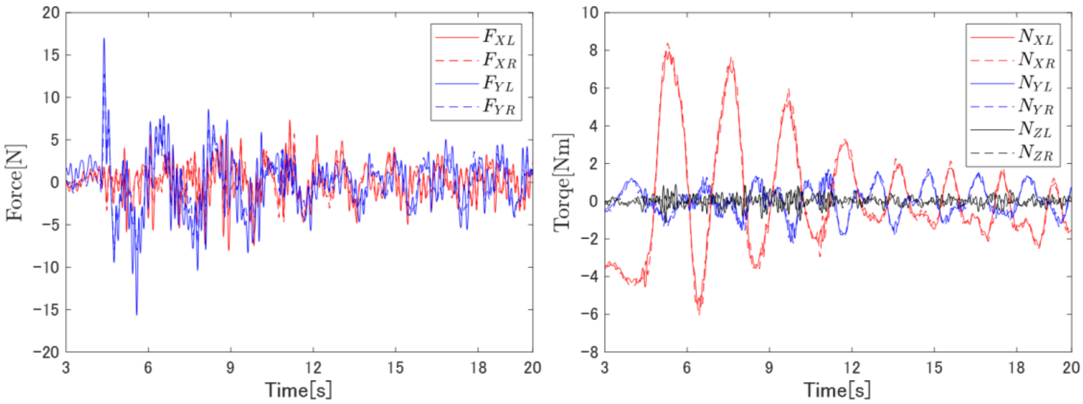

Figure 6. *Left and right sides of time series in Equation (16).*

Table 5. *Maximum error of minimal set of inertial parameters for each link*

	E_{MSj}/M_0[m]	E_{MSj}/m_j[m]	E_{Jj}/I_m[%]	E_{Jj}/J_{mj} [%]
link 0	0.0007	0.0009	0.3657	0.3657
link 1	0.0003	0.0017	0.1006	1.5537

The value of E_{MSj} is maximum error in $\mathbf{MS}_j$ in absolute value and the value of E_{Jj} is maximum error in $\mathbf{J}_j$ in absolute value. The value of I_m is the maximum principal moments of inertia around the origin for attitude 1 and the value of J_{mj} is the maximum principal moment of inertia of the minimum dynamic parameters related to the inertia tensor of each link. Thus, the errors for E_{MSj}/M_0 and E_{Jj}/I_m are comparable to single-body identification accuracy. The error for E_{MSj}/m_j and E_{Jj}/J_{mj} is greater for link 1 than for link 0. This is because the identification error depends on the overall inertial properties. Therefore, the smaller the mass and principal moment of inertia of the link is relative to the mass and principal moment of inertia of the multi-body system, the larger the error will be. However, although the mass of link 1 is less than 20% of the total mass, it can be seen that even the error for the mass and principal moment of inertia of each link is obtained with sufficient accuracy for practical purposes.

From the above results, it is expected that if the mass and principal moment of inertia of the link is too small in relation to the overall mass and principal moment of inertia, the error will be large. However, assuming the case of use in motion simulation, this is unlikely to be a problem as the influence of the small links on the motion simulation is small. The conventional identification methods using a set of measured link motion and ground reaction forces is difficult to apply to the identification of individual human bodies. The proposed identification method is able to identify the minimal set of inertial parameters in all three dimensions from relatively simple movements. Identification accuracy was also found to be very high. As a result, the proposed identification method has the potential to identify the minimal set of inertial parameters with an accuracy that allows individual human differences to be determined.

4 CONCLUSIONS

A new method for identifying the minimal set of inertial parameters of a multi-body system is developed by expanding and applying the identification method based on free vibration measurements, which is the identification method for inertial properties of single-body. The calculation process of the proposed identification method was explained. The performance of the method was evaluated with experimental result using a simple measurement object. The conventional identification methods use a set of measured link motion and ground reaction forces. Only the minimal set of inertial parameters for a sagittal plane can be identified from movements such as walking motion of human bodies. Thus, it is difficult to apply these methods to the identification of individual human bodies. On the other hand, the proposed identification method is able to identify the minimal set of inertial parameters in all three dimensions from relatively simple movements. An accuracy of the proposed method is comparable to single-body identification accuracy. This result is considered accurate enough for practical use and the proposed identification method has the potential to identify the minimal set of inertial parameters with an accuracy that allows individual human differences to be determined.

ACKNOWLEDGMENTS

The authors sincerely thank Resonic Japan Co., Ltd. for helping experiment. Our sincere thanks to Emeritus Professor M. Okuma of Tokyo Tech for his support.

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